ANCIENT GREEK MATHEMATICS: AN UNRIVALLED WESTERN ACHIEVEMENT

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"In the history of civilization the Greeks are preeminent, and in the history of mathematics the Greeks are the supreme event. Though they did borrow from the surrounding civilizations, the Greeks built a civilization of their own which is the most impressive of all civilizations, the most influential in the development of modern Western culture, and decisive in founding mathematics as we understand the subject today. One of the great problems of the history of civilization is how to account for the brilliance and creativity of the ancient Greeks" [Morris Kline, *Mathematical Thought from Ancient to Modern Times* (1972)].

"Of all the manifestations of the Greek genius, none is more impressive and even awe-inspiring than that which is revealed by the history of Greek mathematics" [Sir Thomas Heath, *A History of Greek Mathematics* (1921)].

In this article, I argue that the contribution the ancient Greeks made to mathematics is superior to the *combined* contribution ALL the higher civilizations of the non-Western world made. Mathematics is usually left unmentioned in liberal arts accounts of the "Greek Miracle," even though it had a direct, indispensable role in the rise of modern industrial civilization. We know that mathematics is characterized by rigorous reasoning and precise quantitative calculation; and that it has real-world applications in physics, biology, epidemiology, engineering, chemistry, technologies, computer science, social sciences, and finance. But mathematics is not a mere adjunct to the sciences and technology. Mathematicians have conceived many ideas decades before anyone foresaw their possible applications to science. Without the geometry of Bernhard Riemann, invented in 1854, and other mathematical ideas, the general theory of relativity could not have been articulated. "The revolution in modern physics which began with the work of W. Heisenberg and P. Dirac in 1925," Eric Temple Bell explains in *Mathematics: Queen and Servant of Science* (1931), "could never have started without the necessary mathematics of matrices invented by Cayley in 1858, and elaborated by a small army of mathematicians from then to the present time."

The Greeks constructed an entire geometrical system known as Euclidean, the study of plane and solid figures, about the nature of reality, on the basis of axioms and theorems in a purely deductive manner, without physical evidence, which subsequently found verification and application in the development of modern physics. They were the first people to realize that the universe expresses itself naturally in the language of mathematics, and that mathematical truths have a validity that transcends the limits of time and space. It was this realization about the power of mathematics that persuaded Plato that mathematics comprehends a reality that exists independently of human beings and that mathematicians can apprehend this eternal reality through the sheer power of their deductive reasoning. One does not have to be a Platonist who believes that mathematical truths exist eternally and independently of reality, however, to agree with Adam Smith that mathematical terms express "the most abstract ideas which the human mind is capable of forming," and that it was the Greeks who first conceived a mathematics based on rigorous proofs, which subsequently found experimental applications in Galilean and Newtonian science.

Since prehistoric times, humans have formulated conceptions of number and geometrical forms in creative art. But while there is evidence for the invention of the abstract concept of number, that is, the realization that three sheep, three fingers, and three days, share a common property of "threeness," as well as for the idea of a one-to-one correspondence between the objects of one collection and those of another, including counting an ordered sequence of symbols, such as knots on a cord, basic addition, subtraction, multiplication, and division—most of the cultures of the world made 0 contributions to mathematics defined as a specific field of knowledge, entailing a system of numeration, with a variety of arithmetical calculations with whole numbers and fractions, the solution of linear equations and the mensuration of simple areas and volumes. We only have the Egyptians, Babylonians, Chinese, Indians, and Islamic peoples to compare.

When we compare the Greeks to everyone else, two conclusions are inescapably supported by the best scholarly literature:

1. The Greeks were the first to derive mathematical concepts from pure reasoning alone with little reference to the external world, that is, the first to think about numbers and operations abstractly, as products of the rational powers of man, and to realize that geometry is concerned not with physical objects but with points, lines, triangles, squares, as objects of pure reason. The Greeks invented deductive reasoning, a method wherein reason proposes self-evident premises or axioms from which it deduces theorems in a rigorously consistent manner. The Greeks thus provided proofs for their mathematical derivations, which not even the Indians, the Chinese, and the Muslims, who came after, accomplished at the same level.
2. Although ancient Greek mathematics (600 BC-600 AD) came before Indian (200-1200 AD) and Islamic mathematics (700-1400 AD), with the latter allegedly "picking up the best from Greek and Indian mathematics and developing it further"—in truth the modern European development of analytic geometry, infinitesimal calculus, and the theory of functions, was substantially based on ancient Greek mathematics. Isaac Newton acknowledged the importance of Euclidean geometry in his articulation of his presentation of his laws of motion in the form of two mathematical

theorems: "it's the glory of geometry that from so few principles it can accomplish so much."

"Multicultural Mathematics"

The mathematical achievements of the Greeks, and of modern Europeans who grounded themselves on these achievements, is deeply unsettling to the current effort of the West to become a multicultural place where the diverse races, cultures, and religions of the world are made to feel co-equals in the making of this civilization. The current zeitgeist is that mathematics has been a "global effort…spanning…multiple cultures," or that the achievements of modern Europeans "involved an extensive exchange of ideas among individuals around the world." “Multicultural mathematics” is now a major educational staple of the West. The book, *Multicultural Mathematics: Teaching Mathematics From A Global Perspective*, published in 1991, explicitly states that the “customs, heritage, history, and other aesthetic aspects” of non-European immigrants must be incorporated as “essential components” of “an effective educational program.” The key academic text is *The Crest Of The Peacock: Non-European Roots Of Mathematics*, by George Gheverghese Joseph, professor at University of Manchester. First published in 1991, reprinted 3 times by Penguin, republished by Princeton Press in 2000, with a third edition released in 2011, this book has been cited about a thousand times, with great reviews in prestigious journals. It claims that Europeans scholars have distorted and devalued non-European contributions as “part of the rationale for subjugation and dominance.”

Only two sources are referenced to back this claim: a book published in 1908 by Rouse Ball, and a book by Morris Kline, *Mathematics in Western Culture* (1953). The latter book in particular is faulted for ignoring "a considerable body of research pointing to development of mathematics in Mesopotamia, Egypt, China, pre-Columbian America, India, and the Arab world that had come to light [by the time Kline wrote his book]." Gheverghese proposes a "new model" of the history of mathematics, in which multiple cultures are shown to have played equally significant roles with “cross-fertilization between different mathematical traditions” happening at various times. He flaunts his model as complex, cosmopolitan, and nuanced—superior to the simplistic, linear, one sided, parochial Eurocentric model. Both the old and the new research contradict and invalidate these claims. First, most of the books Gheverghese relies upon to construct his new model are authored by Europeans themselves.

Second, the book by Rouse Ball is actually very cognizant of the contribution of non-Europeans. The title is, *A Short Account of The History of Mathematics*, and it begins with four sections on "Knowledge of the science of numbers possessed by Egyptians and Phoenicians," and "Greek indebtedness to

Egyptians and Phoenicians." Ball's point is that theoretical-deductive mathematics originated with the Greeks.

Third, Gheverghese complains about the "neglect of Arab contribution to… mathematics" without telling his readers that Rouse Ball's book has two long chapters with the titles: "The Mathematics of the Arabs" and "Introduction of Arabian Works into Europe, 1150-1450," in which he affirms original contributions: "From this rapid sketch it will be seen that the work of the Arabs… in arithmetic, algebra, and trigonometry was of a high order of excellence."

Why would Gheverghese, in what is otherwise a solid book in its effort to bring out the best in non-western mathematics, distort the scholarly contribution of Rouse Ball in this manner? Because academics are committed to multiculturalism, and this ideology allows one to distort the truth, for the sake of fighting "white racism".

Fourth, regarding Kline's book, Gheverghese leaves out the fact that Kline's book is specifically about Western mathematics: "The object of this book is to advance the thesis that mathematics has been a major cultural force in Western civilization."

Fifth, why did Gheverghese ignore so many books published after the 1950s, such as Carl Boyer's *A History of Mathematics* (1968)? This book has chapters dedicated to "Egypt," "Mesopotamia," "China and India," and a chapter with the title, "The Arabic Hegemony." Why did he ignore Kline's subsequent book, *Mathematical Thought from Ancient to Modern Times* (1972), possibly the most authoritative historical survey published so far, which opens with chapters on Mesopotamian and Egyptian mathematics, with an additional chapter on "the Hindus and Arabs?" Every book I have read has chapters on non-Europeans. We can go back to D.E. Smith's two volume work, *History of Mathematics*, published in 1923, to find two opening chapters on non-European contributions, and one chapter plus half of another on "Oriental" mathematics, along with separate sections on Oriental contributions inside all the chapters about European contributions. Smith actually co-authored a book on Japanese mathematics.

The basic arguments that Smith presented in *History of Mathematics* are still valid. He offered an opening chapter on Egypt, Mesopotamia, China, and India, as "pioneers in mathematical development before 1000 BC." Then chapters on the contributions of the Classical and the Hellenistic Greeks, from 600 BC to 400 AD. We are informed that during the "five centuries from 500 to 1000 AD… Europe was intellectually dormant," while "there were four or five mathematicians of prominence in India" (p. 152);

and, furthermore, that China saw the greatest accomplishments during the five centuries from 1000 to 1500 AD. While Smith may have neglected Muslim contributors, believing that they were “transmitters of learning rather than creators,” he did offer sections on the “greatest mathematicians” during the Islamic ascendancy from the eight to the fourteenth century. After 1200-1400, Smith's focus shifted back to the Europeans because from this point on all the original ideas came from them alone.

Greeks Compared to Egyptians, Babylonians, and Chinese

The indubitable reality is that, if you look past politicized books on "multicultural mathematics," the scholarly consensus coming from the best books on the history of mathematics for over a century now, and to this day, is that the Greeks, as Kline says in *History of Western Mathematics*, initiated a mathematics that "differed radically from that which preceded them" (p. 24). Dirk Struik's *A Concise History of Mathematics*, in the 1948 edition that I am using, acknowledges Babylonian math "rose to a far higher level than Egyptian… in its computational technique" (p. 23), while arguing that "nowhere" in Babylonian mathematics "do we find any attempt at what we call a demonstration. No argumentation was presented, but only the prescription of certain rules" (p. 31). William Berlinghoff and Fernando Gouvêa follow a similarly developmental interpretation in *Math Through the Ages: A Gentle History for Teachers* (2015). This book, keep in mind, was published by The Mathematical Association of America, which is attuned to the multicultural sensitivities of teachers and students. They point out that, while the Egyptians "could solve simple linear equations… [and] knew how to compute or approximate the areas and volumes of several geometric shapes," Babylonians "made use of extensive tables of products, reciprocals, conversion coefficients, and other constants," and "they could also solve a wide range of problems that we would describe as leading to quadratic equations" (pp. 9-11). However, with the Babylonians, "the ideas behind the methods for solving quadratic equations were probably based on 'cut-and-paste geometry' [and] Babylonian geometry was devoted mainly to measurement…. The Greek mathematicians were unique in putting logical reasoning and proof at the center of the subject" (pp. 11-15).

Stuart Hollingdale, in his biographical book, *Makers of Mathematics* (1989) agrees that "the concept of proof [in Babylonia] is conspicuous by its absence; and there is no clear distinction between exact and approximate results" (p. 11). Carl Boyer's text, *A History of Mathematics*, the revised version co-authored by Uta Merzbach (1989), is unequivocal in its assessment that "pre-Hellenic peoples had no concept of proof, nor any feeling of the need for proof… there are no explicit statements from the pre-Hellenic period that would indicate a felt need for proofs or a concern for questions of logical principles" (p. 47).

In contrast, as Reviel Netz tells us in *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History* (1999), Greek mathematics produced knowledge of general validity; not only about the particular right triangle ABC of the diagram, for example, but about all right triangles. In the polemical culture that Greeks inhabited, where intellectuals sought to outdo each other, where no one could claim control over what constituted the truth on mere utilitarian grounds, or reliance on their official capacities, sage-like status, or adherence to norms sanctified by kinship, making the most persuasive arguments was very important, and in deductive mathematics the Greeks saw the possibility of expressing incontrovertible truths.

But what about the Chinese with their long history, past ancient times, and their "greatest" mathematicians who lived during the Sung Era (960–1279)? "Chinese mathematical works… are in the spirit of the Babylonians rather than the Greeks. They consist of collections of specific problems and present a curious mixture of the primitive and the sophisticated." In the course of their long history, the Chinese became "more advanced than the Babylonians in that they gave general rules, often with formal proofs" (Hollingdale: p. 93), and excellent mathematicians "flourished during the twelfth, thirteenth and fourteenth centuries" (p. 94), when the Europeans were experiencing their "long interlude" in the Middle Ages merely rediscovering the Greek works. Nevertheless, as Joseph Needham admitted, despite his enormous admiration for the overall accomplishments of the Chinese, "Greek mathematics was on a higher level, if only on account of its more abstract and systematic character." In Chinese math, "there was an absence of the idea of rigorous proof." In fact, the Chinese never developed a formal logic. "Mathematics in China was therefore utilitarian… Of mathematics for the sake of mathematics there was very little" (1995: pp. 62-64).

Boyer and Merzbach agree that the ancient Chinese were "repeating the old custom of the Babylonians and Egyptians of compiling sets of specific problems," in contrast to the Greeks of this period who were "composing logically ordered and systematically expository treatises" (p. 222). Zhu Shijie (1249–1314) was "the last and greatest of the Sung mathematicians" but he was a lone, wandering scholar about whom little is known, author of two treatises, of which the first, *Introduction to Mathematical Studies* (*Suanxue qimeng*), was a "relatively elementary work" which was "lost until it reappeared in the nineteenth century." His greater work, *Jade Mirror of the Four Origins* (*Siyuan yujian*), which also "disappeared in the eighteenth century, only to be rediscovered in the next century," represents "the peak in the development of Chinese algebra, for it deals with simultaneous equations and with equations of degrees as high as fourteenth" (pp. 229-30). We will see below, however, that symbolic algebra was a product of early modern Europe, and that Greek mathematics was directly responsible for the development of modern mathematics. This is the reason Morris Kline's text, *Mathematical*

Thought from Ancient to Modern Times, ignores Chinese mathematics in preference for the contributions of Hindu and Arabic mathematics to modern Europe.

The Greeks Were the Forerunners of Modern Mathematics

While reasonable people will have no problems agreeing that Greek mathematics stood far above Egyptian and Babylonian accomplishments, many will find the additional claim that in ancient times the Greeks were already more accomplished than the combined civilizations of China, India, and Islam to be an exaggeration bordering on historical falsification. How could a small population in ancient Greece accomplish more than civilizations that lasted thousands of years, with China and India practicing mathematics after ancient Greece was gone, and Islam standing on the shoulders of Greek achievements? This makes no sense. The most popular argument taught to students today, which prevails online, even though it is not supported by serious scholarly research, is that Hindu-Islamic mathematicians, standing above the Greek legacy, nurtured the rise of modern European mathematics. Here's the World's #1 Online Encyclopedia: "Through contact with other cultures, and especially the absorption of Arab ideas and innovations, European learning in fields such as mathematics was able to go beyond the work of ancient scholars. New fields of study unknown to the Greeks were opened, leading to such developments as the calculus of Newton (1642-1727) and Leibniz (1646-1716), which would revolutionize both mathematics and science."

The impression they want to convey seems reasonable enough: ancient Greek mathematics came before Indian (200-1200 AD) and Islamic mathematics (700-1400 AD), with the latter "picking up the best from Greek and Indian mathematics and developing it further." The best historical scholarship shows, however, that the modern development of analytic geometry, infinitesimal calculus, and the theory of functions, was substantially based on ancient Greek mathematics. This same scholarship acknowledges that the modern West owes a debt to i) "Islamic scholars who collected, preserved, and translated the Greek mathematical texts" (Hollingdale: p. 101), and ii) the Hindu "creation of the decimal positional number system that is universally used today," that is, separate symbols for the numbers 1 to 9, negative numbers, and the notation for a missing position, that is, a zero symbol (Hollingdale: p. 101; Kline, 1972: pp. 183-197).

What about the argument that, while the Greeks originated deductive geometry, the Hindus and Muslims added substantially new ideas to arithmetic leading to the rise of modern symbolic algebra and to trigonometry? Muhammad ibn Musa al-Khwarizmi (780–850) is thus eulogized as the “father of algebra", offering "the first" systematic solution of linear and quadratic equations. Wikipedia informs impressionable students that he is "the first to treat algebra as an independent discipline." Furthermore, Al-Biruni (973–1050), we are told, was among those "who laid the foundation for modern trigonometry," which allowed Muslims to take Greek geometry to higher heights, since trigonometry studies relationships between side lengths and angles of triangles.

First, in response, we need to be aware that ancient Greek mathematics extended from 600 BC to 500 AD, which equals about 1100 years of history. It is commonly assumed that Greece's greatness was restricted to the "Classical" period of the 5th and 4th centuries BC, the age of Socrates, Plato, Aristotle, Aeschylus, Sophocles, Euripides, Hippocrates, Herodotus, Thucydides, the defeat of the Persians, the rise of Athens, the birth of democratic citizenship, and so on. They forget the "Hellenistic" period between the death of Alexander the Great in 323 BC and the rise of Augustus in Rome in 31 BC, and the fact that Greek high culture remained dominant through Roman times. The Classical Period is known as the "Golden Age"—but not in mathematics. The golden age of Greek science was during the Hellenistic era, and, within this era, the golden age of mathematics was from about 300 BC to 200 BC, the time of the three greatest mathematicians: Euclid, Archimedes, and Apollonius. There were many other great mathematicians before and after this age. The birth of mathematics in Greece is generally identified with Thales (623 –545 BC), about whom Aristotle said: "To Thales… the primary question was not What do we know, but How do we know it." Among the things he is said to have proven is that "the pairs of vertical angles formed by two intersecting lines are equal." The next great figure is Pythagoras (580-500 BC) who founded a very influential school, the first to classify numbers: real numbers, rational and irrational, integers, rational fractions, algebraic irrational numbers and transcendental numbers.

The list of mathematicians and their achievements is too long: Archytas (b. 428 BC), Hippasus (400 BC), Hippias (b. 460), Hippocrates of Chios (430, not to be confused with the "father of medicine"), Zeno of Elea (450), Anaxagoras (428), Democritus (460), and the greatest of the Classical Period, Eudoxus (b. 408 BC), known as the father of mathematical astronomy, and the first to formulate the method of exhaustion, which some see as a precursor to the methods of calculus. Menaechmus (380–320 BC) is known for his discovery of conic sections and his solution to the long-standing problem of doubling the cube using the parabola and hyperbola. These men wrote books, some of which have been lost, though we have commentaries on them and some of the titles; for example, Democritus wrote: *On Numbers*, *On Geometry*, *On Tangencies*, *On Mappings* and *On Irrationals*.

After the "golden age" of Euclid, Archimedes, and Apollonius, we have more greats: Aristarchus

(310-230 BC), who wrote *On the Sizes and Distances of the Sun and Moon*; Eratosthenes, remembered for his almost accurate measurement of the Earth; Hipparchus (b. 180 BC), the father of trigonometry; Menelaus (100 AD); Ptolemy (100 –170 AD), the founder of Cartography and Geography, and author of the famous Almagest. Heron (62 AD) is best known for this formula: If a, b, and c are the lengths of the sides: Area = Square root of $\sqrt{s(s - a)(s - b)(s - c)}$ where s is half the perimeter, or $(a + b + c)/2$. We could go on with Diophantus (b. 200AD), author of a series of books called *Arithmetica*, which is seen as the "highest point of Alexandrian algebra," with its "most striking feature" being the solution of indeterminate algebraic equations (Kline, 1972: pp. 138-43). The last of the greats is Pappus (b. 290 AD), known for his Collection (c. 340), and his hexagon theorem in projective geometry, the full significance of which "was not realized until the seventeenth century" (Hollingdale: p. 90).

Archimedes (b. 287 BC) is consistently "ranked with Newton and Gauss as one of the supreme mathematical geniuses of all time" (Hollingdale: p. 64). Suffice it to list his writings that are preserved in full: *On the Equilibrium of Plane Figures, Quadrature of the Parabola, On the Sphere of the Cylinder, On Spirals, On Conoids and Spheroid, On Floating Bodies, The Measurement of the Circle, The Sandreckoner* and *The Method*. *The Conics* by Apollonius is known as a "masterpiece" containing 487 propositions proven by the "rigorous deductive methods characteristic of the Greek masters" (Hollingdale: p. 57). Before I address the role of Hindu-Muslim algebra, I will close with a few words about Euclid. His book, *The Elements*, has been "by far the most influential work ever written," matched only by the Bible. Copernicus, Galileo, Kepler and Newton all built their theories on the basis of Euclidean geometry. *The Elements*, which Bertrand Russell said was "one of the best books ever written," compiles, organizes, and reworks many of the mathematical concepts of Euclid's predecessors into a consistent whole. Its deductive method has been the most important procedure used by Westerners for demonstrating scientific certitude ("truth") until the seventeenth century. No book in the non-west provided such a self-conscious presentation of what it means for a statement to be "known to be true." It states that there must be some set of statements, called axioms, that are assumed to be true intuitively, from which point one can derive other basic statements or theorems. Some have said that a book entitled, *Aryabhatiya*, written in 499 AD by the Indian mathematician Aryabhata, is "somewhat akin to that of Euclid's *Elements*" in that both are "summaries of earlier developments, compiled by a single author." But as Boyer and Merbach point out: "There are, however, more striking differences than similarities, between the two works. *The Elements* is a well ordered synthesis of pure mathematics with a high degree of abstraction, a clear logical structure, and an obvious pedagogical inclination; the *Aryabhatiya* is a brief descriptive work" (p. 237).

Now, it is true, it was in the field of geometry, not arithmetic, that the Greeks constructed their

Euclidean deductive method. In Greek arithmetic operations, which did include algebra, there is no "explicit deductive structure." These are the words of Morris Kline, who goes on to say: "The work of Heron, Nichomachus, and Diophantus, and of Archimedes as far as his arithmetic is concerned, reads like the procedural texts of Egyptians and Babylonians, which tell us how to do things. The deductive, orderly proof of Euclid, Apollonius, and of Archimedes' geometry is gone. The problems are inductive in spirit, in that they show methods for concrete problems that presumably apply to general classes whose extent is not specified" (1972: p. 144).

Kline, however, is less impressed by the achievements of Indians and Muslims in Arithmetic: "The high period [of Indian mathematics] may be roughly dated from about AD 200 to 1200." "Hindu mathematics became significant only after it was influenced by Greek achievements…. The geometry of the Hindus was certainly Greek…. Geometry during this period showed no notable advances…. They did have a special gift for arithmetic." They gave "rules for the multiplication, division, and square roots of irrational expressions…. They used abbreviations of words and a few symbols to describe operations…, The Hindus recognized that quadratic equations have two roots and included negative roots as well as irrational roots…. In indeterminate equations the Hindus advanced beyond Diophantus…. In trigonometry the Hindus made a few advances." However, "the Hindus were less sophisticated than the Greeks in that they failed to see the logical difficulties involved in the concept of irrational numbers. Their interest in calculation caused them to overlook philosophical distinctions, or distinctions based on principles that in Greek thought were fundamental" (1972: 183-90). Moreover, by about 1200, "scientific activity in India declined and progress in mathematics ceased…. It is fairly certain that the Hindus did not appreciate the significance of their own contributions. The few good ideas they had, such as separate symbols for numbers 1 to 9, the conversion to base 10, and negative numbers, were introduced casually with no realization that they were valuable innovations."

Regarding Islamic mathematics, Kline has this to say: "The cultural resources available to the Arabs were considerable. They invited Hindu scientists to settle in Baghdad." Fundamentally, what "the Arabs possessed was Greek knowledge…. In arithmetic the Arabs took one step backward… they rejected negative numbers…. To algebra the Arabs contributed first of all the name. The word 'algebra' comes from a book written in 830." They did not invent algebra: their algebra is "founded on Hindu and also Babylonian and Greek influences…. Arabic geometry was influenced mainly by Euclid, Archimedes, and Heron." In conclusion: "The Arabs made no significant advance in mathematics. What they did was absorb Greek and Hindu mathematics, preserve it, and, ultimately, transmit it to Europe/"

This view is corroborated by the authors I have cited thus far. For example, Berlinghoff and Gouvêa say of Indian mathematics that "the main thing that is mostly missing from their texts is any explanation of how their methods and results were found. They did not give proofs or derivations" (p. 28). Boyer and Merzbach highlight the accomplishments of Brahmagupta (598–668 CE) for being the first to "give a general solution of the linear Diophantine equation ax + by = c, where a, b, and c are integers," and they add that "the trigonometry of the sine function came presumably from India." though Kline thinks that in trigonometry the Hindus made only "a few minor advances" (1972: p. 189). Overall, in the estimation of Boyer and Merzbach, there was a "lack of nice distinction on the part of Hindu mathematicians between exact and inexact results." In their view, indeed, "analytic geometry and calculus had Greek rather than Indian roots, and European algebra came from Islamic countries rather than India" (245-50). According to Hollingdale, the period of Arab pre-eminence between the 9th-11th centuries, only "saw many useful—but not outstanding—advances in algebra, number theory, trigonometry, optics, and, to a lesser extent, geometry" (p. 101).

When all is said, for all the contributions made by Indians and Muslims, it would be Europeans in the modern era, *on the direct strength of what the Greeks had accomplished*, who would transform arithmetic/algebra into proper sciences by introducing symbolism and making "extensive and impressive contributions to the theory of numbers," and thus learning to justify algebraic reasoning, by viewing algebra as an extension of logic in treating quantity, and by reversing the dependence of algebra on geometry, and indeed using algebra to help solve geometric constructions problems. When the Greco-Roman world ended in the sixth century, and Islam took the Mediterranean world, only a small part of the Greek mathematical corpus was known in Europe—until the 11th when scholars from Europe went to Islamic Spain to translate into Latin the works that Muslim scholars had preserved and commented upon. For some time, until about 1400, European mathematics benefitted from this Islamic legacy with its adoption of Hindu numerals. Through the 12th and 13th, Kline writes, Europeans "energetically sought out copies of the Greek works, their Arabic versions, and texts written by Arabs," while contributing their own translations of Greek works into Latin rather than relying on translations that had passed through Arabic translations.

We should not forget, however, that this absorption of Islamic mathematics occurred within an emerging rationalist Christian framework, the "Renaissance of the 12th Century," which included the invention of universities with a "rational" curriculum and a continuous sequence of scholastic philosophers. I will mention only a few names: Roger (not Francis) Bacon (1220–1292) is identified as beginning experimental science and for writing about the importance of mathematics to all science; and Jean Buridan and Nicholas Oresme are both acknowledged for their demonstration that "the effective velocity in uniformly difform motion was the average of the initial and final velocities." It is even said that Oresme anticipated Descartes' coordinate geometry, with "contributions towards the development of the concept of graphing functions and approaches to investigating infinite series."

This broader rationalist atmosphere, together with the rise of universities, was absent in the Islamic world, despite its admiration for Aristotle. Only Christians would seek to provide logical proofs for the existence of God. Spectacles and mechanical clocks were both invented in 13th century Europe. Romanesque and Gothic architecture required more practical geometry than the architecture of other civilizations. For the sake of modesty, however, let us say that, up until about 1500, European mathematicians, "with their algebraic emphasis, derived more inspiration from Arabic and medieval mathematics than from the much richer inheritance of Classical Greece" (Hollingdale: p. 107). It still remains the case that the European breakthrough into modern mathematics that came in the 1600s was primarily grounded in Greek mathematics.

Before this breakthrough there was Leonardo Pisa, also known as Fibonacci (1175-1250), identified as "the most creative mathematician of the medieval Christian world," who followed Islamic mathematicians "in using words rather than symbols and in basing the algebra on arithmetical methods/" His work, *De practica geometrie* (1225), however, was based on Euclid's book. Nicolas Chuquet's *Le Triparty en la science des nombres* (1484) explained the Hindu-Arabic number system and how to perform arithmetic with this system. This treatise was novel, however, in devising an exponential notation where the power of the unknown was indicated by an exponent; and in presenting an algebraic notation with an isolated negative number, though he viewed negative numbers as absurd. Girolamo Cardono "astonished" the mathematical world by giving algebraic solutions to both cubic and quartic equations in his book, *Ars magna* (1545).

After other prominent names, the most significant transition to modern mathematics came with the introduction of a fully symbolic algebra by François Viète [Franciscus Vieta] (1540-1603). Because Hindus and Muslims had placed their practical arithmetical calculations in the forefront of their mathematics, and had elevated algebra on an arithmetic rather than a geometric basis; and because the European transition to modern mathematics took place in arithmetic and algebra, it is commonly believed that Hindu and Islamic mathematics laid the groundwork for Vieta's transition to algebraic symbolism, and subsequent developments in analytic geometry, calculus, and functions. Not true. Vieta's book, *In artem analyticem isagoge* [*Introduction to the Art of Analysis*] (1591), was part of his "program of rediscovering the method of analysis used by the ancient Greek mathematicians," as *The*

Britannica Guide to the History of Mathematics recognizes (2011: p. 96). His algebra had a firm Greek geometrical foundation. "His aim was to uncover and restore the algebraic relationships that were, he believed, hidden behind the geometrical presentations of the Greek masters" (Hollingdale: p. 120). Vieta saw his new symbolic algebraic method "as an advancement over the ancient method, a view he arrived at by comparing the geometric analysis contained in Book VII of Pappus's *Collection*, with the arithmetic analysis of Diophantus's *Arithmetica*." (*The Britannica Guide*: p. 96). Despite the attempt of this *Guide* to portray mathematics as a "global effort…spanning…multiple cultures," it can't hide the actual truth. Once this Guide hits the modern era, not a single non-European is mentioned since none participated in modern mathematics.

Vieta was the first to use algebraic symbols or letters deliberately and systematically, not only to represent unknown quantities but also as general coefficients. "The Arabs had not advanced one iota in symbolic notation." The "turning point in the history of algebra" came with Vieta (Dantzig, [1930] 1954: 85-7). Before him, in Europe, letters had been used for the unknown, and the first abbreviations used from the 1400s on were p for plus and m for minus; the = was introduced in 1557 by Robert Recorde. These changes in notation, the use of special words and number symbols, were essentially abbreviations of normal words. In fact, prior to Vieta, it was only Diophantus (AD 200) who had consciously introduced symbolism to make algebraic writing more compact and efficient. Vieta's education was overwhelmingly based on the writings of the ancient Greeks—Apollonius, Archimedes, Pappus, Diophantus; and the works of European mathematicians such as Cardano, Tartaglia, and Stevin. After Vieta, his analytic algebra was applied to the study of curves by his French countrymen Fermat and Descartes, who were also motivated by the same goal of applying new algebraic techniques to Greek geometry, leading to the development of analytic geometry. Vieta actually drew a conceptual line between his new symbolic algebra and arithmetic, calling the former a true algebra, with the potential to become a universal science. In other words, the arithmetic algebra of the Hindus and Muslims was not, in his estimation, truly algebraic.

The Mathematics of Perspective

This transition to modern mathematics was founded primarily on the Greek achievement, not as a mere intellectual exercise, but in response to the newly emerging scientific world of the Renaissance era, the age of exploration and the rise of Copernican astronomy. Copernicus's heliocentric system, and Kepler's (1571-1630) three planetary laws, were based on the Platonic belief that the universe was ordered according to a mathematical plan and that the truths of nature could be revealed in mathematical laws and geometrical terms—ideas that were absent in both the Islamic and the Hindu world. Renaissance perspective painting, the realistic depiction of scenes on canvas by incorporating three-dimensions, relative distances, size, and positions of objects, was likewise based on a thorough study of Euclidean geometry. The European cartographic revolution, the mapping of the world, was intimately connected to Greek mathematics; Gerard Mercator's (1512-94) map solved the problem of projecting figures from a sphere onto a flat surface.

The advantage deductive geometry had over practical arithmetic, trial and error, or reasoning by induction and analogy, is that its validity came from the logical derivation of conclusions from self-evident premises, rather than from approximate inferences based on observations of empirical facts restricted to a time and place. Even if we were to argue that deductive mathematics is merely a conventional language that Westerners imposed upon the world, rather than an accurate revelation of the structure of the universe, the success of Euclidean mathematical models lay precisely in mimicking or predicting the behavior of physical bodies. In the ideal world of abstraction that Galileo created, without resistance or friction, in which physical bodies were reduced to geometrical forms, perfectly smooth bodies moving on a perfect plane, the principles of Euclidean geometry held. As Galileo declared, "the grand book of the universe…cannot be understood unless one first learns to comprehend the language and to read the alphabet in which it is composed…the language of mathematics." It was the Greeks who discovered the language of nature.

Ricardo Duchesne has written a number of articles on Western uniqueness. He the author of The Uniqueness of Western Civilization, Faustian Man in a Multicultural Age, Canada in Decay: Mass Immigration, Diversity, and the Ethnocide of Euro-Canadians.

Featured: A folio from Synagogue (Collection) *by Pappus, ca. 10th century AD.*