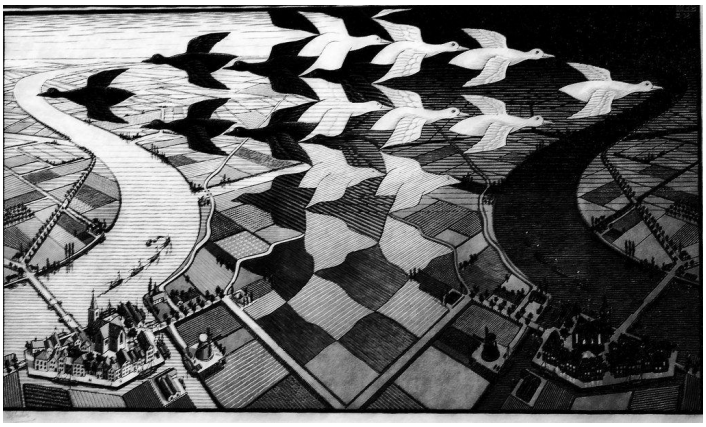

THE WEST CONCEPTUALIZED ALL PARADOXES

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Defining Paradoxes

That Europeans conceptualized all the paradoxes in history, with a minimal contribution by the Chinese, testifies to a major contrast in the intellectual trajectories of civilizations. As R.M. Sainsbury writes, "paradoxes are fun," but "unlike puzzles and teasers, which are also fun, paradoxes raise serious problems." Sainsbury defines a paradox as "an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises." Patrick Hughes and George Brecht tell us that paradoxical propositions may be described as

i. self-referential, in which the statement refers to itself;

ii. contradictory, in which the statement is false and true at the same time;

iii. and circular in which the statement is characterized by a vicious circle, or infinite regress.

Some believe that most paradoxes, whether they are semantic, set-theoretic, or epistemic, share the same underlying self-referential structure. Consider the famous Liar's paradox: "This sentence is false." Trying to determine whether this sentence is true or false leads to a contradiction. If we conclude that the sentence is true, then it cannot be true. If we agree that the sentence is false, then the sentence is true. Either answer leads to a contradiction, or a vicious circle. This paradox is self-referential in that the sentence is talking directly about itself.

What makes the Liar's paradox more than a witty remark or a sophism is that both of the two contradictory conclusions were obtained by seemingly rigorous reasoning based on apparently sound premises. However, paradoxes come in different degrees of difficulty; some paradoxes, Sainsbury says, are "weak or shallow," based on unfounded suppositions, faulty reasoning, or ostensibly vague wording. The famous Barber paradox, he thinks, is "not very deep." This paradox asks who shaves the barber if the barber shaves all and only those villagers who do not shave themselves. This paradox makes the supposition that such a barber exists even though such a barber or such a village does not exist.

The Western mind came up with many paradoxes for which they attempted solutions—because this mind has a peculiar inclination to seek the truth according to its own self-legislated rational capacities,

rather than in acquiescence to kinship norms or theocratic mandates. It is a most dynamic mind that came up with three fundamental laws of proper reasoning:

1. the law of contradiction, which states that a proposition cannot be both true and false;
2. the law of excluded middle, which states that for every proposition, either this proposition or its negation is true; and
3. the law of identity, which states that each thing is identical with itself.

For the Western mind, if a claim is illogically inconsistent, in violation of these laws, then the claim or the reasoning behind it must be rejected.

Europeans took Zeno's paradoxes seriously, for they seem to suggest that one could reach a logically unacceptable conclusion on the basis of sound reasoning from apparently sound premises. They wondered whether these paradoxes revealed deficiencies in the way we reason, calling for improvements in our reasoning powers, a better system of logic and a more precise usage of language. For example, Russell's paradox (1901) of a set that includes all and only those sets that do not contain themselves as members, encouraged or was itself part of an investigation into the realm of the foundations of logic and the philosophy of mathematics, which revealed errors in classical logic that were instrumental in the development of modern logic and set theory. With this new logic, and the insights of physics and contemporary mathematics, many paradoxes have found a solution.

Reasoning Through the Contradictory Character of Reality

We would be mistaken, however, to assume that all European thinkers came to view paradoxes as mere expressions of faulty reasoning calling for a more logically perfect language, in which equivocation could not arise because all the ambiguity of everyday words was removed. Without denying the classic laws of logic, and agreeing that sentences cannot admit of self-nullifying contradictions, some Europeans came to the view that the nature of reality is contradictory, and that the human mind is limited in its capacity to offer rationally consistent answers about the ultimate questions of the universe and life. Heraclitus came to the conclusion that reality was inherently contradictory and thus paradoxical. Other European thinkers encountered complex riddles, conundrums, and puzzles for

which they believed there were no rationally justified solutions. The ultimate questions of reality were inherently unanswerable or beyond rational solution or dogmatic certainty. Sextus Empiricus (160-210 AD) believed that all belief systems were ultimately grounded in dogmatic premises or criteria for which any attempt at their justification inevitably led to an infinite regress. Roy Sorensen's *A Brief History of the Paradox: Philosophy and the Labyrinths of the Mind* (2003) is the best book in the English language about the many great European thinkers who have grappled for millennia with the most puzzling philosophical conundrums.

Immanuel Kant famously presented the four "antinomies of pure reason" as demonstrations of reason's inability to reach completeness on the most fundamental questions of the universe. He believed one could give equally justified answers for

i. the thesis that the universe has no beginning, and the antithesis that the universe has a beginning;

ii. the thesis that every composite substance in the world is made up of simples, and the antithesis that no composite substance in the world is made up of simples;

iii. the thesis that there is freedom in the world, and the antithesis that everything is ruled by necessity;

iv. the thesis that a necessary being is either part or cause of the world, and the antithesis that a necessary being is neither part nor cause of the world.

The rational inclination of the European mind should not be equated with arrogance. Kant believed that reason could come up with two impeccably solid arguments against the infinity of the past and against the idea that the past has a finite beginning, by rigorously showing how each of these positions leads to an equally absurd conclusion; and how each position is no less justified than the other. The thesis that there is no beginning, or an infinite past, leads to the absurd conclusion that we had to wait an infinite time to get to the present, and since we are living in the present, it follows that the past is finite with a beginning in time. But the antithesis that there is a beginning leads to the befuddling conclusion that there must have been a preceding time in which there was no world and no time, which leads to the conclusion that the past is infinite.

For Hegel, however, the very recognition by the mind that there are limits beyond which reason can't provide answers is a demonstration that cognition understands its limits, and, in this respect, is capable of going beyond those limits, by realizing that the coexistence of opposites is inherent to the nature of reality: an antinomy is not, therefore, an imperfection or defect of the mind, but a demonstration that every determination in the actual concrete world is a unity of opposing contradictions. Hegel's dialectical logic was an attempt to provide a new way of thinking through the contradictory nature of things. "Identity is merely the determination of the simple immediate, of dead being; but contradiction is the root of all movement and vitality; it is only insofar as something has a contradiction within it that it moves, has an urge and activity."

While many contemporary logicians believe that Zeno's paradoxes have been solved by transfinite arithmetic, originated by Georg Cantor in the late 1800s, and that there are logically consistent ways to "solve" all paradoxes, some have come to the neo-Hegelian view known as "dialetheism," which proposes that some paradoxical inconsistencies or contradictions can be accepted without incoherence, in the sense that both "A" and "not-A" can both be true. Dialetheism is associated with the development of a paraconsistent logic to deal with sentences in which both its affirmation and its negation are true.

Lower Degree of Sophistication of Chinese Paradoxes

The only other civilization to have articulated and debated paradoxes is China. During the Warring States period (479-221 BC), a group of scholars identified with the "School of Names" (*Míngjiā*) relished in the use of paradoxical and puzzling expressions, and in the discussion of the semantic relations between words and the world they pointed to. The intellectual culture of paradoxes in China, however, was fundamentally different from that of the West, in their degree of sophistication, the reaction of intellectuals to them, and the absence of philosophical reflections about the contradictory nature of the universe. The School of Names remained an isolated moment in China's intellectual history. The Confucians in control of intellectual discussions dismissed the paradoxical expressions of the School of Names as "bizarre expressions" that discouraged young minds from the proper use of language and the obligation of educated gentlemen to promote "ritual propriety and righteousness."

With the exception of brief texts attributed to Gōngsūn Lóng, very little first-hand knowledge of the figures associated with this School survived. What we know about Chinese paradoxes is mostly based on the *Xunzi*, which is an ancient Chinese collection of philosophical writings attributed to Xunzi (Xun

Kuang), a 3rd-century BC philosopher associated with the Confucian tradition, who condemned paradoxical expressions as "frivolous." While members of the Mohist School (479–221 BC) summarized many of the paradoxes of the School of Names, they did so only to offer counter-arguments against what they perceived to be sophistical ideas inconsistent with the commonsense use of language.

The current academic Chen Bo makes a strong effort to show that the School of Names reached the same level of logical sophistication and use of abstract universals as the ancient Greeks, or at least "comparable to Greek civilization to some extent." But while Chen sometimes even tries to show that some Chinese paradoxes exhibited the abstract "concepts of class, kind, and membership" found in Russell's paradox about classes discovered in 1901, in the end Chen barely shows that Chinese paradoxes reached the same degree of abstraction as Zeno's famous paradoxes. He basically acknowledges this in his decision to define paradoxes in "quite a broad way, almost including all the fallacies, sophisms, puzzles, riddles, and paradoxes [which were] quite influential" in China. As Chris Fraser recognizes, there are expressions categorized as "paradoxes" that are best described as mere philosophical statements about the nature of reality, such as: "The ultimately great has no outside, call it the Great One. The ultimately small has no inside, call it the Small One." Or, "Universally care for the myriad things. Heaven and earth are one body."

Chen confounds his otherwise good analysis of Chinese paradoxes when he tries to argue that the mere utterance by Chinese philosophers of statements about whether "there is really an external world independently of us," or "do we really know the states of external things," or "do we really know other minds and their states"—actually constitute paradoxes, rather than basic epistemological and ontological questions. Chen is on stronger ground showing that the following Chinese paradoxes are "quite close to Zeno's paradoxes of motion and infinity":

1. "A Wheel does not touch the ground."

2. "The shadow of a flying bird has never moved."
3. "A one-foot stick has taken away half of its length every day, it will still not be exhausted after ten thousand generations."

For example, number three resembles Zeno's dichotomy paradox. This Chinese paradox says that a finite-long stick can be cut into infinitely many sections since half of the length of the stick will remain

no matter how many times you cut it. Zeno's dichotomy paradox states that you will never reach a finite destination no matter how fast and how long you run since any given distance that you try to cover will consist of infinitely divisible distances.

Chen admits, however, that from the Chinese version of this paradox we can only get the concept of infinite divisibility, not the concept that all motion is an illusion. Chris Fraser correctly points out that another Chinese paradox—"The barbed arrow at its swiftest, there is time when it neither moves nor stops"—which has been likened to Zeno's paradox of the arrow, is actually different in that the Chinese arrow is neither in motion nor at rest, whereas Zeno's arrow is at rest in every instant of time and does not move. Zeno invented his paradoxes in order to demonstrate through rigorous reasoning that any kind of change was impossible and that reality was indivisible, despite appearances to the contrary.

Fraser further notes that the "steps in the reasoning" behind the articulation of Chinese paradoxes are almost entirely based on second-hand accounts by critics of the Schools of Names, in which the reasoning about their meaning "remains confusing and the justification for them murky." While we know Zeno's paradoxes second-hand through Aristotle's writings, we do so through the highly rigorous logical mind of Aristotle. Rather than dismissing them as sophistical, Aristotle tried to find a solution to Zeno's paradoxes. Zeno's paradoxes raised serious philosophical problems in the Western tradition because they were seen to be based on sound reasoning and thus to constitute a challenge to the claims of reason about its ability to understand the nature of things. The solutions offered in contemporary times have come from mathematics, modern logic, and physics.

Moreover, whereas Europeans would come up with numerous paradoxes, some of which came to be associated with crises in European thought and with revolutionary advances in logic, in China the tradition of the School of Names dissipated, never to be improved upon. Thus, the paradoxes remained rather simple; basically, their prevailing theme, as Fraser notes, was that the distinctions we observe in the world are "not inherently fixed but relative to a standpoint" or scale. For example, take the paradox "The sky is as low as earth, mountains are level with marches." It simply says that by the scale of the Great One, the difference between the height of the sky and earth, mountains and marches, are insignificant. Another so-called paradox—"Eggs have feathers"—merely states that potential feathers are already possessed by the egg.

The most discussed Chinese paradox is one attributed to Gōngsūn Lóng that simply states: "The white horse is not [a] horse" (*báimǎ fēi mǎ*). An entire discourse developed around this paradox. The Mohist

School in particular sought to counter it with the commonsense idea that words do represent or signify things and that language should be used to communicate by appealing to commonly shared use of language, for distinguishing similar and different kinds of things. Yet, interesting as this discourse was in generating sophisticated semantic discussions, this statement seems to lack a true paradoxical nature. It seems to be a semantic claim that insofar as the terms "white horses" and "horses" denote distinct features about horses, it follows that white horses and horses cannot be equated. Critics argue that it is simply a statement that refuses to distinguish the true claim that the term "white horse" is not identical with "horse" from the false claim that the less general term "white horse" is not part of the more general term "horse."

Cheng Bo tries to argue that Chinese paradoxes use abstract universals, terms that can be instantiated or exemplified by many particular things, much like the ancient Greeks and Europeans in the modern era. The academic Fung Yu Lan claims that the Chinese terms *mǎ* "horse" and *bái* "white" are used to designate abstract concepts—"horseness" and "whiteness." But Chad Hansen thinks that the grammatical structure of the ancient Chinese language was such that it lacked abstract entities, such as ideas and concepts. Chinese nouns are "mass nouns," or nouns without a plural form, which do not inflect for abstraction and plurality. Thus, it seems that the best way to understand why the Chinese were unable to conceive paradoxes with a high degree of sophistication is that their mind barely reached what Jean Piaget identified as the "formal operational stage of cognition." It remained stuck at the "concrete operational stage," barely reaching the formal stage, until Westerners taught them how to think in deductive ways.

The Concrete-Operational Chinese Mind

Georg W. Oesterdiefkhoff, one of the foremost Piagetian theorists in the world, is convinced that many years of cross-cultural empirical research by Jean Piaget and his followers have demonstrated that the stages of mental development of children and adolescents (from sensorimotor, through preoperational and concrete operations, to formal operations) reflect the stages of cognitive evolution "humankind" has gone through from primitive, ancient and medieval, to modern societies. The cognitive processes of humanity have not always been the same, but have improved over time. Viewing Piaget's theory as more than a theory about the cognitive development of children, Oesterdiefkhoff has elaborated it into a grand theory covering the cognitive experience of all peoples throughout history, from primitive peoples with a preoperational mind (characteristic of children ages 2–4), to agrarian peoples with a concrete operational mind (characteristic of children ages 6–12), to modern peoples with a formal

operational mind (age 12 and up).

Oesterdiefkhoff observes that "thousands of empirical studies across all continents and social milieus, from the 1930s to the present" have been conducted demonstrating that, depending on the level of cultural scientific education, the nations of the world in the course of history can be identified as preoperational, concrete operational, and formal operational. The formal operational stage entails a capacity to think about abstract relationships and symbols in the absence of concrete examples, that is, a capacity to grasp syllogistic reasoning with hypothetical premises that may not refer to real objects, including a capacity to formulate explanatory hypotheses.

According to studies conducted in the 1960s and 1970s, even educated adults living in Papua New Guinea did not reach the formal stage. Australian Aborigines who were still living a traditional lifestyle barely developed beyond a preoperational stage in their adult years. As Lucien Lévy-Bruhl (1857–1939) had already observed in his *Primitive Mentality* (1923), dreams, divination, incantations, sacrifices, and omens, not inferential reasoning and objective causal relations, are the phantasmagorical doors through which primitives get access to the intentions and plans of the unseen spirits they believe control natural events. Adults in premodern civilizations, like ancient China, exhibit the concrete operational stage of thinking in the way they apprehend length, volume, time, space, weight, area, and geometric qualities. It is not that adults in primitive and premodern cultures are similar to children in modern cultures in their emotional development, experience, and ability to survive in a hostile environment. It is that the reasoning abilities of adults in pre-modern civilizations are restricted to what they apprehend at the perceptual level and are bound up with the sensory appearances of the world, barely transcending appearances and context-bound experiences through the development of hypothetical and abstract reasoning.

The abilities associated with the first two Piagetian stages (e.g., control over motor actions, walking, mental representation of external stimuli, verbal communication, ability to manipulate concepts involving concrete objects), have been acquired universally by humans since prehistoric times. In this sense, they can be called "biologically primary" qualities that children across cultures accomplish at the ages and in the sequence more or less elucidated by Piaget. They are universal abilities built into human nature, ready to unfold with little educational socialization. While the concrete-operational abilities of stage three (e.g., the "ability to conserve" quantities—e.g., to know that the same quantity of a liquid remains when the liquid is poured into a differently shaped container) are either lacking in primitive cultures or emerge at later ages in children than they do in modern cultures; these abilities

may also be described as biologically primary insofar as they have emerged in all advanced agrarian civilizations.

It is also the case that in modern societies all individuals with a primary education acquire concrete operational abilities. The abilities required in the first three stages can thus be identified as "culturally universal." Only formal operations cannot be said to be endogenously generated. The skills associated with this stage (inductive logic, hypothesis testing, reasoning about proportions, combinations, probabilities and correlations) are not cognitive skills bound to emerge in all literate civilizations. They are highly specific skills that a significant proportion of the population, even in modern Western cultures, fails to acquire. The abilities required in formal operational thinking are better described as biologically secondary abilities.

While the current Chinese population, after the arrival of Western modernization, has clearly reached the formal operational stage of reasoning, this was not the case in ancient China. In contrast, formal operational reasoning was clearly visible among the educated elite in ancient Greece. We learn in Reviel Netz's, *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History*, that Greek mathematics produced knowledge of general validity; not only about the particular right triangle ABC of the diagram, for example, but about all right triangles. This formal operational trait—this ability to think about numbers and geometric relationships in a purely abstract way—is what makes Greek mathematics historically novel in comparison to all preceding concrete operational mathematics. This formal operational mind is amply visible in Aristotle's magisterial works of logic, in the rise of theoretical geometry in ancient Greece, in the invention of trigonometry by Hipparchus, in Archimedes' work on hydro-statics and the mechanics of pulleys and levers, in Eratosthenes and his calculations to determine the circumference of the earth. It is visible as well in the hypothetical-deductive form of Euclid's *Elements*, in which circles, right angles, parallel lines are explicitly defined in terms of a few fundamental abstract entities, such as points, lines, and planes, on the basis of which many other propositions (theorems) are deduced. And in Ptolemy's *Almagest*, which postulates epicycles, eccentric circles, and equant points, with the latter being imaginary points in space from which uniform circular motion is measured.

Joseph Needham, a great admirer of Chinese intellectual history, recognizes that "strong" as Chinese algebra was, it was "utilitarian" and "concrete," "devoted to the [practical] problems ruling officials had to solve." Unlike "the predilection of Greek science and mathematics for the abstract, the deductive and the pure over the concrete," Chinese mathematics lacked "the idea of rigorous proof" but inhabited "a

mental outlook which avoided the development of formal logic…and which allowed associative or organic thinking to dominate" (p. 62-3). So, in light of these intellectual realities, here's a short list of the many paradoxes conceived by Europeans:

Zeno's Racetrack Paradox
Paradox of the Heap
Newcomb's Trolley Paradox
Prisoner's Dilemma
Paradox of the Ravens
Russell's Paradox
Leonard Euler's Paradox
The Liar
Achilles and the Tortoise
The Arrow
Sorites Paradox
Forrester's Paradox
Class Paradoxes

Ricardo Duchesne has also written on the creation of the university. He the author of The Uniqueness of Western Civilization, Faustian Man in a Multicultural Age, Canada in Decay: Mass Immigration, Diversity, and the Ethnocide of Euro-Canadians.

Featured image: "Day and night," by Maurits Cornelis Escher, 1938.